FINAL MATH 265

Evaluation out of 100. Calculators are NOT allowed.

- 1. (8) Find the volume of the solid enclosed by the parabolic cylinders $y = 1 x^2$, $y = x^2 1$ and the planes x + y + z = 2, 2x + 2y z + 10 = 0.
- 2. (8) Use a double integral to find the area of the region within both of the circles $r = \sin \theta$ and $r = \cos \theta$ ((r, θ)) are polar coordinates).
- 3. (8) The joint density function for a pair of random variables X and Y is

$$f(x,y) = Cx(1+y)$$
 if $0 \le x \le 1$, $0 \le y \le 2$

and f(x,y) = 0 otherwise. Find a) the value of constant C; b) $P(X \le 1, Y \le 1)$; c) $P(X+Y \le 1)$

- 4. (8) Find the volume enclosed by the torus $\rho = \sin \varphi$ ((ρ, θ, φ) are spherical coordinates).
- 5. (8) Find the volume and centroid of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.
- 6. (8) Evaluate

$$\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2+y^2+z^2} dz dx dy$$

- 7. (8) Evaluate $\iint_R \cos\left\{\frac{y-x}{y+x}\right\} dA$ using an appropriate change of variables, where R is the trapezoidal region with vertices (1, 0), (2, 0), (0, 2) and (0, 1).
- 8. (8) Find the work done by he force field $\mathbf{F}(x, y, z) = (y + z)\mathbf{i} + (x + z)\mathbf{j} + (x + y)\mathbf{k}$ on a particle that moves along the line segment from (1, 0, 0) to (3, 4, 2).
- 9. (8) Find the area of the helicoid $\mathbf{r}(u,v) = u\cos v\mathbf{i} + u\sin v\mathbf{j} + v\mathbf{k}, 0 \le u \le 1, 0 \le v \le \pi$
- 10. (8) Evaluate the surface integral $\iint_S \mathbf{F} d\mathbf{S}$ if $F = x\mathbf{i} + y\mathbf{j} + 2z\mathbf{k}$ and S is the surface of the solid bounded by hemisphere $x^2 + y^2 + z^2 \le a^2$, $z \ge 0$ and the plane z = 0 with outward orientation.
- 11. (8) Evaluate $\iint_S \operatorname{curl} \mathbf{F} d\mathbf{S}$ where

$$\mathbf{F}(x, y, z) = x^2 y z \mathbf{i} + y z^2 \mathbf{j} + z^3 e^{xy} \mathbf{k}$$

and S is the part of the sphere $x^2 + y^2 + z^2 = 5$ that lies above the plane z = 1 and S is oriented upwards.

12. (8) Solve the initial value problem

$$y'' + y' - 2y = x + \sin 2x$$
, $y(0) = 1$, $y'(0) = 0$.

13. (6) A force of 400N stretches a spring 2 meters. A mass of 50 kilogramms is attached to the end of the spring and released from the equilibrum position with an upward velocity of 10m/s. Find position of the mass as function of time.

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