Mast 218 Sec A Final Exam December 2007

Professor: *Richard Hall*
Instructions: *Please answer all 5 questions. Explain your working carefully. Calculators are permitted.*

1. Consider the integral $f(x) = \int_0^x \frac{dt}{1+t^3}$
   (a) Find a Taylor series about $x = 0$ for $f(x)$.
   (b) What is the interval of convergence of this Taylor series.
   (c) Find a Taylor polynomial $T(x)$ to approximate $f(x)$ with error $< 10^{-6}$ for $0 \le x \le \frac{1}{2}$. Explain!

2. Consider the curve in $\Re^3$ given by $\mathbf{r}(t) = \left(t^2/2, \cos(t), \sin(t)\right), \quad t \in \Re$.
   (a) Find the unit tangent vector $\mathbf{T}(t)$.
   (b) Find the curvature $\kappa(t)$.
   (c) For $t = 0$, find the triad of unit vectors, that is to say, the unit tangent vector $\mathbf{T}$, the principal normal $\mathbf{N}$, and the binormal $\mathbf{B}$.
   (d) Find the tangent line which touches the curve at $t = 0$.

3. Consider the surface $S: \ z = 9 - (x^2 + 4y^2)$.
   (a) Sketch the surface S, and describe precisely the level curve $z = 0$.
   (b) Find the tangent plane to S at $(x, y) = (2, 1)$.
   (c) Find the directional derivative $D_{\mathbf{u}} f$ at $(x, y) = (2, 1)$, where the direction vector $\mathbf{u}$ is given by $\mathbf{u} = (3, -2)$.

4. Consider the function $W(x, y) = e^{st}(s + t + r^2)$, where $r = x + y$, $s = 2y - x$, and $t = x - y$.
   (a) Find expressions for the partial derivatives $\partial W/\partial x$ and $\partial W/\partial y$.
   (b) Find the tangent plane to the surface $z = W(x, y)$ at $(r, s, t) = (3, 0, 1)$
   (c) Use part (b) to estimate the numerical value of $W(1.9, 1.1)$.

5. Consider the function $z = f(s, t) = (1 + s)(1 + t)(s + t)$.
   (a) Find the critical points of $f(s, t)$ for $(s, t) \in \Re^2$.
   (b) Classify the critical points as min, max, or saddle points.
   (c) What is the minimum value of z ?