Math 264 Secs A & B Final Exam December 2010

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Instructions: *Please answer all 5 questions which carry equal marks.*
Explain your working carefully.
Calculators are permitted. [Lined booklets]

1. Consider the sine integral $S(x) = \int\limits_0^x \frac{\sin(t)}{t} dt$.
   (a) Assume that the integrand $\sin(t)/t$ is continuous at $t = 0$, and sketch its graph.
   (b) Find a Taylor series about $x = 0$ for $S(x)$.
   (c) What is the interval of convergence of this Taylor series.
   (d) Find a Taylor polynomial $T(x)$ to approximate $S(x)$ with error $< 10^{-6}$ for $0 \leq x \leq \frac{1}{2}$. Explain!

2. Consider the curve in $\Re^3$ given by $\mathbf{r}(t) = \left\langle \sin(t),\, t^2/2,\, \cos(t) \right\rangle, \quad t \in \Re$.
   (a) Find the unit tangent vector $\mathbf{T}(t)$.
   (b) Find the curvature $\kappa(t)$.
   (c) For $t = 0$, find the triad of unit vectors, that is to say, the unit tangent vector $\mathbf{T}$, the principal normal $\mathbf{N}$, and the binormal $\mathbf{B}$.
   (d) Find the plane which is normal to the curve at $t = \pi/4$.

3. Consider the surface $S: \quad z = f(x, y)$, where $f(x, y) = 16 - (9x^2 + 4y^2)$.
   (a) Sketch the surface S , and describe precisely the level curve $z = 7$.
   (b) Find the tangent plane to S at $(x, y) = (1, 2)$.
   (c) Find the directional derivative $D_{\mathbf{u}}f$ at $(x, y) = (1, 2)$, where the $\mathbf{u}$ is a unit vector in the direction $\langle 3, -4 \rangle$.

4.
   (a) If $z = xe^{y\cos(x)}$, where $x = st$, and $y = s^2 + t^2$, find $\partial z/\partial s$ and $\partial z/\partial t$.
   (b) If $x\tan(z) + e^x \sin(y) + z^2xy = 10$, explain what is meant by $\partial z/\partial x$. Find expressions for $\partial z/\partial x$ and $\partial x/\partial y$.

5. Consider the function $z = f(x, y)$, where $f(x, y) = 2x^2y - 4x^2 - 12y^3 + 18y^2$.
   (a) Find the critical points of $f(x, y)$ for $(x, y) \in \Re^2$.
   (b) Classify these critical points as loc min, loc max, or saddle points.
   (c) Does $f(x, y)$ have a global minimum value? Explain.