Math 264 Secs A & B Final Exam December 2011

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Instructions: *Please answer all 5 questions which carry equal marks.*
Explain your working carefully.
Calculators are permitted. [Lined booklets]

1. Consider the integral $J(x) = \int\limits_0^x \frac{1-\cos(t)}{t^2} dt$.

   (a) Assume that the integrand $f(t) = (1 - \cos(t))/t^2$ is continuous at $t = 0$, and sketch the graph of $f(t)$.

   (b) Find a Taylor series about $x = 0$ for $J(x)$.

   (c) What is the interval of convergence of this Taylor series?

   (d) Find a Taylor polynomial $T(x)$ to approximate $J(x)$ with error $< 10^{-5}$ for $0 \le x \le 1$. Explain!

2. Consider the curve in $\Re^2$ which has the polar equation

$$r(\theta) = \frac{5}{1 - \frac{1}{2}\sin(\theta)}, \quad \theta \in [0, 2\pi].$$

   (a) Find the largest and smallest $r(\theta)$, and the corresponding values of θ .

   (b) Sketch the curve.

   (c) Find the enclosed area.

3. Consider the surface $S:$ $z = f(x, y) = x^2 y^2$.

   (a) Sketch the level curve $z = 16$.

   (b) Find the tangent plane to S at $(x, y) = (1, 2)$.

   (c) Find the directional derivative $D_{\mathbf{u}}f$ at $(x, y) = (-2, 3)$, in the direction defined by $\mathbf{v} = \langle 3, -4 \rangle$.

4. Suppose $w = \frac{x+y}{x+z}$.

   (a) If $x = s + tr$, $y = t + sr$, and $z = r + st$, find $\partial w/\partial s$ and $\partial w/\partial t$ at $(s, t, r) = (1, 2, 3)$.

   (b) If $w = 7$, find an expression for $\partial x/\partial y$ in terms of x , y , and z .

5. Consider the function $z = f(x, y) = x^3 + y^3 - 3x - 12y + 20$.

   (a) Find the critical points of $f(x, y)$ for $(x, y) \in \Re^2$.

   (b) Classify these critical points as loc min, loc max, or saddle points.

   (c) Does $f(x, y)$ have a global minimum value? Explain.