```
(* ------------------------------------------------------ *)
(*     dn.nb   Cauchy integral formula in Mathematica     *)
(* ------------------------------------------------------  *)
(* This is just an entertainment by Richard Hall         *)

(* The path w + a  is circle of radius b and centre a for t in [0, 2Pi] *)

w[t_, b_] := b Exp[I t]

w[t, b]
```

$$b\, e^{i t}$$

```
z[x_, y_] := x + I y

(* Four simple test functions  *)

f1[s_] := s^3

f2[s_] := Exp[-s]

f3[s_] := Exp[-s] s^2

f4[s_] := Sin[s] * s^2

(* Try to avoid complications of pure functions in Mathematica         *)

(* Cauchy formula simplified for a circular path radius b centre a>0  *)
(* we get a numerator factor of w from w'[t] = i w[t]                 *)
(* Thus dn finds the nth derivative of f(z) at z = a  by integration  *)

dn[f_, a_, n_] :=
 FullSimplify[(n! / (2 Pi)) Integrate[f[w[t, b] + a] / w[t, b]^n, {t, 0, 2 Pi},
    Assumptions → {b > 0, a > 0}]]

dn[f1, a, 1]
```

$$3\, a^2$$

```
dn[f1, a, 2]
```

$$6\, a$$

```
dn[f1, a, 4]
```

$$0$$

```
dn[f2, a, 1]
```

$$-\text{Cosh}[a] + \text{Sinh}[a]$$

```
dn[f2, a, 2]
```

$$e^{-a}$$

```
dn[f3, a, 2]
```

$$(2 + (-4 + a)\, a)\, e^{-a}$$

```
(* check directly *)

f3''[a]
```

$$2\, e^{-a} - 4\, a\, e^{-a} + a^2\, e^{-a}$$

```
dn[f4, a, 1]
```

```
a (a Cos[a] + 2 Sin[a])
```