

# Department of Mathematics & Statistics

## Math 366 & 601J Sec AA Final Exam December 2013

**Professor:** *Richard Hall*

**Instructions:** *Please answer all 5 questions which carry equal marks.  
Explain your work carefully.  
Approved calculators are permitted.*

1. Find all values of the expressions:

(a)  $(4\sqrt{3} + 4i)^{\frac{1}{3}}$ .

(b)  $5^i$ .

2. Consider the analytic function  $f(z) = u(x, y) + iv(x, y)$ ,

where  $u(x, y) = x^2 - y^2 - x + 1$ .

(a) Find the harmonic conjugate  $v(x, y)$  to the given  $u(x, y)$ .

(b) Sketch the families of level curves  $u(x, y) = c_1$  and  $v(x, y) = c_2$ .

(c) Show that the two sets of curves in (b) are orthogonal.

3. Consider the function  $f(z)$  defined by

$$f(z) = \begin{cases} \frac{e^z - 1}{z}, & z \neq 0 \\ 1, & z = 0. \end{cases}$$

(a) Show that  $f(z)$  may be redefined as an entire function by means of a suitable Taylor (Maclaurin) series.

(b) Find  $f'(z)$

(c) What is the function  $g(z) = \frac{1}{i\pi} \int_{\gamma} \frac{f(s)ds}{(s-z)^3}$ , where  $\gamma$  is a simple closed positively oriented contour with  $z$  in its interior.

(d) Find the first three terms of a power series in  $z$  for the function  $1/f(z)$ .

- 
4. By means of a suitable contour integral, evaluate the real improper integral  $I$  given by

$$I = \int_0^{\infty} \frac{x^2 dx}{(x^2 + 9)(x^2 + 25)}.$$

5. Consider the family of integrals

$$J_n(a, b) = \int_0^{2\pi} \frac{d\theta}{(a + b \cos \theta)^n}, \quad a > b > 0, \quad n = 1, 2, 3 \dots$$

- (a) Use contour integration to find an expression for  $J_1(a, b)$ .  
(b) Use part (a) to find  $J_2(a, b)$ .  
HINT: consider  $\frac{\partial}{\partial a} J_1(a, b)$ .  
(c) Provide a recipe for  $J_n(a, b)$ .

---

The present document and the contents thereof are the property and copyright of the professor(s) who prepared this exam at Concordia University. No part of the present document may be used for any purpose other than research or teaching purposes at Concordia University. Furthermore, no part of the present document may be sold, reproduced, republished or re-disseminated in any manner or form without the prior written permission of its owner and copyright holder.

---