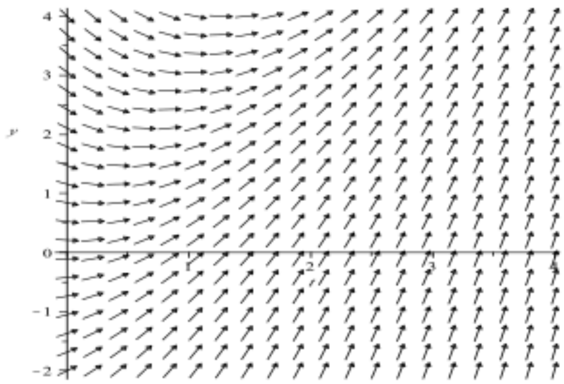
Math 370 Assignment 1
Sec. 2.1

9.(a)



(b) All solutions eventually have positive slopes, and hence increase without bound.

(c) The integrating factor is $\mu(t) = e^{\int (1/2)\,dt} = e^{t/2}$. The differential equation can be written as $e^{t/2}y' + e^{t/2}y/2 = 3t\,e^{t/2}/2$, that is, $(e^{t/2}\,y/2)' = 3t\,e^{t/2}/2$. Integration of both sides of the equation results in the general solution $y(t) = 3t - 6 + c\,e^{-t/2}$. All solutions approach the specific solution $y_0(t) = 3t - 6$.

14. The integrating factor is $\mu(t) = e^{2t}$. After multiplying both sides by $\mu(t)$, the equation can be written as $(e^{2t}\,y)' = t$. Integrating both sides of the equation results in the general solution $y(t) = t^2 e^{-2t}/2 + c\,e^{-2t}$. Invoking the specified condition, we require that $e^{-2}/2 + c\,e^{-2} = 0$. Hence $c = -1/2$, and the solution to the initial value problem is $y(t) = (t^2 - 1)e^{-2t}/2$.

19. After writing the equation in standard form, we find that the integrating factor is $\mu(t) = e^{\int (4/t)\,dt} = t^4$. Multiplying both sides by $\mu(t)$, the equation can be written as $(t^4\,y)' = t\,e^{-t}$. Integrating both sides results in $t^4 y(t) = -(t+1)e^{-t} + c$. Letting $t = -1$ and setting the value equal to zero gives $c = 0$. Hence the specific solution of the initial value problem is $y(t) = -(t^{-3} + t^{-4})e^{-t}$.

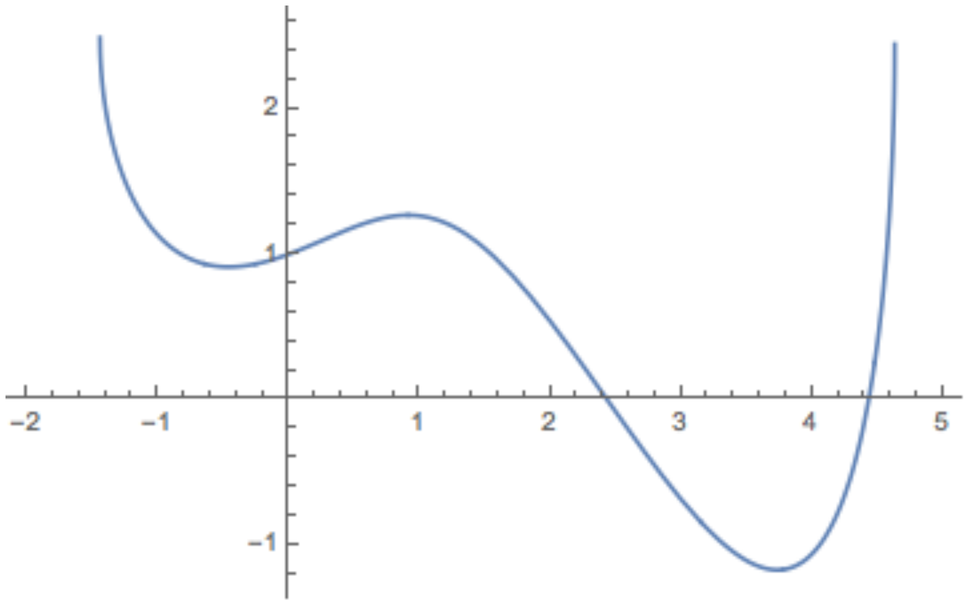
Sec. 2.2

(17) $(2y - 5)dy = (3x^2 - e^x)dx$

Integrating, we find $y^2 - 5y = x^3 - e^x + C$.

IC $y(0) = 1$ imply $C = -3$. Hence $y = 5/2 \pm (x^3 - e^x + 13/4)^{1/2}$;

IC ➔(-) . From the sqrt, or the graph, $-1.44451 < x < 4.62971$.



(21)

Integrating and fitting the IC $y(0) = 1$, we find $y^3 - 3y^2 = x + x^3 - 2$.

From the differential equation we see $y' =$ infinity when $y = 0$ or $y = 2$,

That is to say, $x = 1$ or -1. We want the solution through $x = 0$. Thus $|x| < 1$.

(23)

The de and the IC $y(0) = 1$ imply $2x + x^2/2 = 1 - 1/y$.

We want the solution including $y(0) = 1$.

If we complete the square in x, we obtain $y = 1/(3 - (x+2)^2/2)$.

Thus avoiding the singularity at $6 = (x+2)^2$, that is $x = \sqrt{6} - 2$,

But including $y(0) = 1$, we need the left branch of the graph shown.

The minimum value over this branch is $y(-2) = 1/3$.