Math 370 Assignment 3
Sec. 2.5

15.(a) Inverting Eq.(11), Eq.(13) shows t as a function of the population y and the carrying capacity K. With $y_0 = K/3$,

$$t = -\frac{1}{r}\ln\left|\frac{(1/3)\,[1-(y/K)]}{(y/K)\,[1-(1/3)]}\right|.$$

Setting $y = 2y_0$,

$$\tau = -\frac{1}{r}\ln\left|\frac{(1/3)\,[1-(2/3)]}{(2/3)\,[1-(1/3)]}\right|.$$

That is, $\tau = (\ln 4)/r$. If $r = 0.025$ per year, $\tau \approx 55.45$ years.

(b) In Eq.(13), set $y_0/K = \alpha$ and $y/K = \beta$. As a result, we obtain

$$T = -\frac{1}{r}\ln\left|\frac{\alpha\,[1-\beta]}{\beta\,[1-\alpha]}\right|.$$

Given $\alpha = 0.1$, $\beta = 0.9$ and $r = 0.025$ per year, $\tau \approx 175.78$ years.

Sec 2.6

4. First divide both sides by $(2xy+2)$. We now have $M(x,y) = y$ and $N(x,y) = x$. Since $M_y = N_x = 0$, the resulting equation is exact. Integrating M with respect to x, while holding y constant, results in $\psi(x,y) = xy + h(y)$. Differentiating with respect to y, $\psi_y = x + h'(y)$. Setting $\psi_y = N$, we find that $h'(y) = 0$, and hence $h(y) = 0$ is acceptable. Therefore the solution is defined implicitly as $xy = c$. Note that if $xy + 1 = 0$, the equation is trivially satisfied.

16. $M(x,y) = y\,e^{2xy} + x$ and $N(x,y) = bx\,e^{2xy}$. Note that $M_y = e^{2xy} + 2xy\,e^{2xy}$, and $N_x = b\,e^{2xy} + 2bxy\,e^{2xy}$. The given equation is exact, as long as $b = 1$. Integrating N with respect to y, while holding x constant, results in $\psi(x,y) = e^{2xy}/2 + h(x)$. Now differentiating with respect to x, $\psi_x = y\,e^{2xy} + h'(x)$. Setting $\psi_x = M$, we find that $h'(x) = x$, and hence $h(x) = x^2/2$. We conclude that $\psi(x,y) = e^{2xy}/2 + x^2/2$. Hence the solution is given implicitly as $e^{2xy} + x^2 = c$.

Sec 2.6

30. The given equation is not exact, since $N_x - M_y = 8x^3/y^3 + 6/y^2$. But note that $(N_x - M_y)/M = 2/y$ is a function of y alone, and hence there is an integrating factor $\mu = \mu(y)$. Solving the equation $\mu' = (2/y)\mu$, an integrating factor is $\mu(y) = y^2$. Now rewrite the differential equation as $(4x^3 + 3y)dx + (3x + 4y^3)dy = 0$. By inspection, $\psi(x, y) = x^4 + 3xy + y^4$, and the solution of the given equation is defined implicitly by $x^4 + 3xy + y^4 = c$.

Sec. 2.7

14. The Euler formula is $y_{n+1} = (1 - ht_n)y_n + hy_n^3/10$, with $(t_0, y_0) = (0, 1)$.

(a) 0.950517, 0.687550, 0.369188, 0.145990, 0.0421429, 0.00872877

(b) 0.938298, 0.672145, 0.362640, 0.147659, 0.0454100, 0.0104931

(c) 0.932253, 0.664778, 0.359567, 0.148416, 0.0469514, 0.0113722

(d) 0.928649, 0.660463, 0.357783, 0.148848, 0.0478492, 0.0118978