Math 370 Assignment 4

Sec. 3.2

(41) Straightforward

45. $P = x^2$, $Q = x$, $R = -1$. We have $P'' - Q' + R = 0$. The equation is exact. Write the equation as $(x^2y')' - (xy)' = 0$. After integration, we conclude that $x^2y' - xy = c$. Divide both sides of the differential equation by x^2. The resulting equation is linear, with integrating factor $\mu = 1/x$. Hence $(y/x)' = cx^{-3}$. The solution is $y(t) = c_1 x^{-1} + c_2 x$.

Sec. 3.3

42. The equation transforms into $y'' + 6y' + 10y = 0$. The characteristic roots are $r = -3 \pm i$. The solution is

$$y = c_1 e^{-3x} \cos(x) + c_2 e^{-3x} \sin(x) = c_1 \frac{1}{t^3} \cos(\ln t) + c_2 \frac{1}{t^3} \sin(\ln t).$$

Sec. 3.4

20.(a) The characteristic equation is $r^2 + 2a\,r + a^2 = (r + a)^2 = 0$.

(b) With $p(t) = 2a$, Abel's Formula becomes $W(y_1, y_2) = c\,e^{-\int 2a\,dt} = c\,e^{-2at}$.

(c) $y_1(t) = e^{-at}$ is a solution. From part (b), with $c = 1$, $e^{-at}\,y_2'(t) + a\,e^{-at} y_2(t) = e^{-2at}$, which can be written as $(e^{at}\,y_2(t))' = 1$, resulting in $e^{at}\,y_2(t) = t$.

Sec. 3.5

23.(a) The characteristic equation for the homogeneous problem is $r^2 - 5r + 6 = 0$, with roots $r = 2, 3$. Hence $y_c(t) = c_1 e^{2t} + c_2 e^{3t}$. Consider $g_1(t) = e^{2t}(3t + 4)\sin t$, and $g_2(t) = e^t \cos 2t$. Based on the form of these functions on the right hand side of the ODE, set $Y_2(t) = e^t(A_1 \cos 2t + A_2 \sin 2t)$ and $Y_1(t) = (B_1 + B_2 t)e^{2t} \sin t + (C_1 + C_2 t)e^{2t} \cos t$.

(b) Substitution into the equation and comparing the coefficients results in

$$Y(t) = -\frac{1}{20}(e^t \cos 2t + 3e^t \sin 2t) + \frac{3}{2} t e^{2t}(\cos t - \sin t) + e^{2t}(\frac{1}{2} \cos t - 5 \sin t).$$

Sec. 3.6

7. The functions $y_1(t) = e^{-2t}$ and $y_2(t) = te^{-2t}$ form a fundamental set of solutions. The Wronskian of these functions is $W(y_1, y_2) = e^{-4t}$. The particular solution is given by $Y(t) = u_1(t)\,y_1(t) + u_2(t)\,y_2(t)$, in which

$$u_1(t) = -\int \frac{te^{-2t}(t^{-2}e^{-2t})}{W(t)}dt = -\ln t \quad \text{and} \quad u_2(t) = \int \frac{e^{-2t}(t^{-2}e^{-2t})}{W(t)}dt = -1/t.$$

Hence the particular solution is $Y(t) = -e^{-2t}\ln t - e^{-2t}$. Since the second term is a solution of the homogeneous equation, the general solution is given by

$$y(t) = c_1 e^{-2t} + c_2 te^{-2t} - e^{-2t}\ln t.$$