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euler1.nb

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example y' = y(3 - xy), y(0) = 1/2.

We first solve analytically (with capricious help from Matheematica).

This is a Bernouilli equation with y^2, so we define z = y^(-1) and immediately find
z' + 3z = x, z(0) = 2.

In[98]:=
```
DSolve[z'[x] + 3 z[x] == x, z[x], x]
```

Out[98]=
$$\left\{\left\{z[x] \to -\frac{1}{9} + \frac{x}{3} + e^{-3x} C[1]\right\}\right\}$$

In[99]:=
```
{{z[x] → -1/9 + x/3 + e^(-3 x) C[1]}}
```

Out[99]=
$$\left\{\left\{z[x] \to -\frac{1}{9} + \frac{x}{3} + e^{-3x} C[1]\right\}\right\}$$

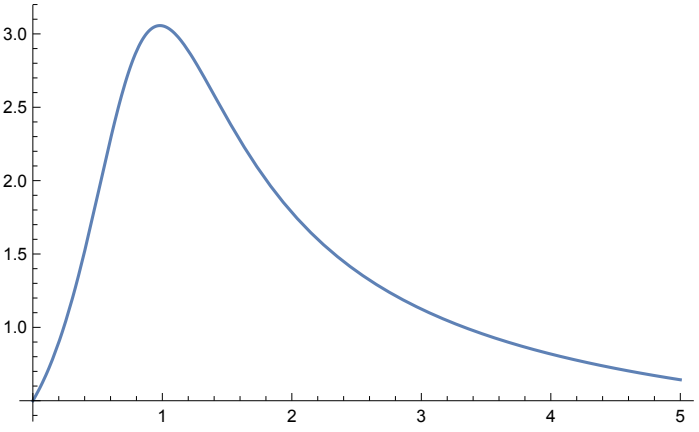
In[100]:=
```
y[x_] := (-1/9 + x/3 + c e^(-3 x))^(-1)
```

In[101]:=
```
c = Last[Solve[y[0] == 1 / 2, c]]
```

Out[101]=
$$\frac{19}{9}$$

In[102]:=
```
Plot[y[x], {x, 0, 5}]
```

Out[102]=



Now we solve the problem numerically by using Euler's first method,
namely for y' = f(x,y), we define Yn = approximation for y(nh), where
h is the fixed step size, and n = 0,1,2,3 ... Since we start from x=0, Xn = nh.
Thus in our problem Y0 = y[0] = 1/2 and Yn+1 = Yn + h f(Xn,Yn).
More specifically we have Yn+1 = Yn +hYn(3-XnYn) , with Y0 = 1/2.

In[103]:=
```
ynum[n_, h_] := Module[{ya = Array[0, n + 1]}, ya[[1]] = 1 / 2;
  Do[ya[[i + 1]] = ya[[i]] (1 + h * (3 - i * h * ya[[i]])), {i, 1, n}]; ya]
```

In[109]:=
```
yna = ynum[10, 0.1]
```

Out[109]=
$$\left\{\frac{1}{2}, 0.6475, 0.833365, 1.06254, 1.33614,\right.$$
$$\left. 1.64772, 1.97914, 2.29869, 2.56558, 2.74285, 2.81339\right\}$$

```
(* compare *)
```

In[108]:=
```
y[1.]
```

Out[108]=
```
3.05504
```

In[124]:=
```
(* compare graphs:  analytic = lower (blue)  *)

(*  Now use h = 0.025    *)
```

In[120]:=
```
yna = ynum[40 , 0.025]
```

Out[120]=
$$\left\{\frac{1}{2}, 0.537344, 0.577284, 0.619955, 0.665491, 0.714019, 0.765658, 0.820518,\right.$$
$$0.87869, 0.940249, 1.00524, 1.07369, 1.14557, 1.22082, 1.29934, 1.38097,$$
$$1.46547, 1.55256, 1.64189, 1.73301, 1.82545, 1.91862, 2.0119, 2.10461, 2.19601,$$
$$2.28536, 2.37189, 2.45485, 2.5335, 2.60718, 2.67527, 2.73724, 2.79269,$$
$$\left. 2.84128, 2.88283, 2.91724, 2.94456, 2.96489, 2.97848, 2.98563, 2.9867\right\}$$

In[123]:=
```
ListPlot[{Table[{(i - 1) * 0.025, yna[[i]]}, {i, 1, 41}],
  Table[{i * 0.025, y[i * 0.025]}, {i, 0, 40}]}]
```

Out[123]=