Mast 330 /Math 370 Midterm Test 27 October 2004

Professor:	*Richard Hall*
Instructions:	*Please answer all 4 questions.*
	Explain your work clearly. Calculators are permitted.

1. Solve the initial-value problem $y' = e^{-y} \cosh(x)$, $x \geq 0$, $y(0) = 1$, and find the value of $y(5)$. [Recall: $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$].

2. Find the general solution to the following differential equation and also a particular solution satisfying $y(\pi) = 1$:

$$xy' + 2y = \frac{\cos(x)}{x}, \quad x > 0.$$

3. Consider the differential equation

$$2x\,dx + (2y + x^2 + y^2)dy = 0.$$

   (a) Show that the equation is *not* exact as it stands but can me made exact by use of a suitable integrating factor μ. Find μ.

   (b) Find the general solution of the equation, and also a particular solution satisfying $y(0) = 2$.

4. Consider the following differential equation which describes the vibrations of a spring-mass system:

$$4y''(t) + 4y'(t) + y(t) = 0, \quad t \geq 0.$$

   (a) Find the general solution.

   (b) Find a particular solution satisfying the initial conditions $y(0) = 1$, $y'(0) = -2$ and provide a rough sketch of the graph of $y(t)$ for $t \in [0, 20]$. When is $y = 0$?