

Mast 330 /Math 370 Midterm Test 24 October 2005

Professor: *Richard Hall*

Instructions: *Please answer all 4 questions.*

Explain your work clearly. Calculators are permitted.

1. Solve the initial-value problem $y' = (1 + y^2) e^{-x}$, $x \geq 0$, $y(0) = 3$,

2. Consider the differential equation

$$(e^x(1+x)y - 2xy^2) dx + (2xe^x - 3x^2y) dy = 0$$

(a) Show that the equation is *not* exact as it stands but can be made exact by use of a suitable integrating factor μ . Find μ .

(b) Find the general solution of the equation, and also a particular solution satisfying $y(1) = 2e$.

3. Find the general solution to the following differential equation

$$y' + \frac{1}{x}y = y^4, \quad x > 0.$$

4. Consider the following differential equation which describes the vibrations of a spring-mass system:

$$y''(t) + 2y'(t) + 2y(t) = 0, \quad t \geq 0.$$

(a) Find the general solution.

(b) Find a particular solution satisfying the initial conditions

$y(0) = 2$, $y'(0) = 0$ and provide a rough sketch of the graph of $y(t)$ for $t \in [0, 10]$.