# Math 370 Midterm Test $27^{th}$ October 2016

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**Professor:** *Richard Hall*
**Instructions:** *Please answer all 4 questions.*
*Explain your work clearly. Calculators are permitted.*

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1. Solve the initial-value problem $(1+x^2)\,y' = (4+y^2)\,3x$, $y(0) = 1$.

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**Solution [1]**: *The de is separable:* $dy/(4+y^2) = 3xdx/(1+x^2)$. *Integration and the application of* tan *yields* $y = 2\tan\left[3\ln(1+x^2)+C\right]$. *The IC imply* $C = \arctan(1/2)$.

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2. Consider the differential equation

$$y' = \frac{x+2y}{y-2x}$$

   (a) Find the general solution for regions where $(y-2x) \neq 0$.

   (b) What happens to the solution curve if $(y-2x)$ approaches zero?

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**Solution [2]**: *There are a number of ways to solve this. (i) As an exact equation* $(x+2y)dx + (2x-y)dy = 0$ *with supposed solution* $f(x,y) = c$. *Then* $f_x = x+2y$ *and* $f = x^2/2 + 2yx + k(y)$, *and from* $f_y = 2x - y$ *we find* $k(y) = -y^2/2$. *The solution is then* $x^2 + 4xy - y^2 = C$. *(ii) By making the substitution* $v = y/x, y' = v + xv'$, *leading to a separable de, (iii) By using polar coordinates, as I did in class.*

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3. Consider the differential equation

$$2y' + xy = 8x.$$

   (a) Find the general solution.

   (b) Find a particular solution satisfying $y(0) = 5$ and provide a rough sketch of this solution.

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**Solution [3]**: *This is first-order linear de for which the standard solution uses the integrating factor* $\mu(x) = e^{\int p dx} = e^{x^2/4}$. *The formula yields* $y(x) = 8 + Ce^{-x^2/4}$. *The IC demands that* $C = -3$ *so that the curve is an upside-down Gaussian hump with a minimum at* $x = 0$ *of* $y = 5$, *and right and left large-*$x$ *limits of* $y = 8$.

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4. Consider the following differential equation in which $y$ and $g$ are functions of $t$:

$$y'' + y' - 6y = g.$$

(a) Find the general solution if $g = 0$.

(b) Find the general solution if $g(t) = 4\sin(t)$.

**Solution** [4]: **(a)** *For $g = 0$, we try the function $y(t) = e^{rt}$ and find $r^2 + r - 6 = 0$, which implies $r = 2$ or $r = -3$. Thus the general solution when $g = 0$ is $y(t) = Ae^{2t} + Be^{-3t}$.* **(b)** *Since $g(t) = 4\sin(t)$ does not resonate with the system, we look for a particular solution of the form $y_P(t) = a\cos(t) + b\sin(t)$. Substituting $y_P$ into the differential equation yields a solution when $\{b - 7a, -7b - a\} = \{0, 4\}$, that is to say $a = -2/25, b = -14/25$. The general solution of the differential equation is therefore $y(t) = Ae^{2t} + Be^{-3t} - \frac{2}{25}[\cos(t) + 7\sin(t)]$.*