Math 473 Assignment 1

Professor: *Richard Hall*
Instructions: *Please explain your solutions carefully.*
Due Date: *19th January 2017.*

1.1 This problem is an entertainment (not graded). The unknown is a function f.

- 1.1a Suppose $f(x)$ is (i) continuously differentiable for all $x \in \Re$, and (ii) the function f satisfies Cauchy's functional equation $f(x+y) = f(x) + f(y)$. Prove that $f(x) = f(1)x$.
- 1.1b Now solve Cauchy's equation under the weaker assumption that $f(x)$ is continuous. HINT: first solve for fractions, then recall that $x_n \to x \Rightarrow f(x_n) \to f(x)$.

1.2 If $y = y(x)$, find the general solution to the ode $y' + 3y = \sin(x)$.

1.3 If $u = u(x, y)$, find the general solution to the pde $u_x = 2x$.

1.4 Find the general solution to the pde $u_x + 3u = \sin(x)$.

1.5 Suppose $u = u(x, y)$ satisfies the pde $3u_x + 4u_y + u = e^{x+y}$.

- (a) Find the general solution.
- (b) Find a particular solution satisfying $u(0, y) = e^y$.

1.6 Suppose $u = u(x, y)$ satisfies the pde $yu_x + u_y = 1$

- (a) Find the general solution.
- (b) Find a particular solution satisfying $u(0, y) = y + y^4$.