Math 473 Solutions for 6.1{2} 6.2 {2, 3, 4, 5a}

2. (*b*) $(xu)_{xx} + (xu)_{yy} = (xu_x + u)_x + xu_{yy} = x(u_{xx} + u_{yy}) + 2u_x = 2u_x$. Thus, if u is harmonic, then xu_x is harmonic iff $u_x = 0$ (i.e., $u(x, y) = f(y)$). Now, $u(x, y) = f(y)$ is harmonic iff $f''(y) = 0$ or $u(x, y) = ay + b$, for constants a and b.

(*c*) Among the many possibilities, consider x and $2x$. More generally, consider x and any harmonic function $u(x, y)$ which is not of the form $ay + b$ (cf. part (*b*)).

Section 6.2

2. (*a*) $u(x, y) = \frac{9}{\sinh\left(\frac{8\pi M}{L}\right)} \sin\left(\frac{8\pi x}{L}\right) \sinh\left(\frac{8\pi(M - y)}{L}\right)$

(*b*) $u(x, y) = \frac{1}{\sinh\left(\frac{\pi M}{L}\right)} \sin\left(\frac{\pi x}{L}\right) \sinh\left(\frac{\pi y}{L}\right)$

(*c*) $u(x, y)$ is the sum of the answers in parts (*a*) and (*b*)

(*d*) $u(x, y)$ is obtained from the answer to (*c*) by interchanging x and y and interchanging L and M.

3. $u(x, y) = \frac{1}{\sinh \pi} (\sinh(\pi - y) \sin x + \sinh y \sin x + \sinh(\pi - x) \sin y + \sinh x \sin y)$.

4. (*a*) $U(x, y) = x - y + 2xy$

(*b*) $u(x, y) = \frac{3}{\sinh \pi} \sin(\pi x) \sinh(\pi - \pi y) + \frac{1}{\sinh 2\pi} \sin(2\pi y) \sinh(2\pi - 2\pi x) + U(x, y)$.

5. (d) Follow Example 4 on p364 of the text.