Math 473 Assignment 3

Section 3.2

2. Follow the hint.

4. Let $f(x) = 5\sin(3x) - 3\sin(5x)$. Then $f'(x) = 15\cos(3x) - 15\cos(5x) = 30 \cdot \sin(4x) \cdot \sin(x) = 0$ when $x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$. Now, $f(0) = f(\pi) = 0$, $f(\frac{\pi}{4}) = f(\frac{3\pi}{4}) = \frac{5}{2}\sqrt{2} + \frac{3}{2}\sqrt{2} = 4\sqrt{2}$, while $f(\frac{\pi}{2}) = -8$. Thus, the Maximum/Minimum Principles yield $-8 \leq u(x,t) \leq 4\sqrt{2}$.

Section 3.3

2. (a) $\frac{1}{L}\int_0^L u(x,t)\,dx = \frac{1}{L}\int_0^L \left(\sum_{n=0}^N a_n e^{-(n\pi/L)^2 kt}\cos(\frac{n\pi x}{L})\right)\,dx$
$= \frac{1}{L}\int_0^L a_0\,dx + \frac{1}{L}\sum_{n=1}^N e^{-(n\pi/L)^2 kt}\sin(\frac{n\pi x}{L})\Big|_0^L = a_0$.
The total heat energy in the insulated rod is constant.
(b) $\int_0^L u_t(x,t)dx = \int_0^L ku_{xx}(x,t)dx = ku_x(x,t)|_0^L = 0$, by the B.C..

3. (a) $c_n \exp\left(-\frac{(n+\frac{1}{2})^2\pi^2 kt}{L^2}\right)\sin\left(\frac{(n+\frac{1}{2})\pi x}{L}\right)$, $n = 0, 1, 2, \ldots$.

(b) $u(x,t) = \sum_{n=0}^N c_n \exp\left(-\frac{(n+\frac{1}{2})^2\pi^2 kt}{L^2}\right)\sin\left(\frac{(n+\frac{1}{2})\pi x}{L}\right)$.

4. $u(x,t) = u_p(x,t) + v(x,t) = x - 1 + e^{-9\pi^2 t/2}\sin(\frac{3\pi x}{2})$.

7. $u(x,t) = 4 - 2\pi + 2x + 7e^{-9t/4}\cos(\frac{3x}{2})$.