```
(* ----------------------------------------------------- *)
(*   t473-17-Q2.nb                                    *)
(* -----------------------------------------------------*)

Clear["Global`*"]

(*  L = a = 1  *)

(* f(x) = 10x/a + 4 , u(x,Infinity) = 10,  hence  g(x) = 10x/a - 6  *)

(* to satisfy the BC we extend f(x) and g(x) to be
   even about x=0, odd about x=1, and with period 4. *)


(* compute the F coefficients as a formula so we do this once only  *)
```

$$c[n\_, a\_] = (2/a) \text{Simplify}[$$
$$\text{Integrate}[(10\,x/a - 6)\,\text{Cos}[(2\,n - 1)\,\text{Pi}\,x/(2\,a)], \{x, 0, a\}], \text{Element}[n, \text{Integers}]]$$

$$-\frac{16\left(5 + (-1)^n\left(-1 + 2\,n\right)\pi\right)}{\left(\pi - 2\,n\,\pi\right)^2}$$

```
(* check the normalization  *)
```

$$(2/a)\,\text{Simplify}[\text{Integrate}[\text{Cos}[n\,\text{Pi}\,x/(2\,a)]\wedge 2, \{x, 0, a\}], \text{Element}[n, \text{Integers}]]$$

```
1

v[x_, t_, a_, n1_] :=
```
$$\text{Sum}[c[n, a]\,\text{Cos}[(2\,n - 1)\,\text{Pi}\,x/(2\,a)] * \text{Exp}[-((2\,n - 1)\,\text{Pi}/(2\,a))\wedge 2\,t], \{n, n1\}]$$

```
(* just the problem patch [0, 1], the FS for g(x) and the first basis fn *)
```
$$\text{Plot}[\{v[x, 0, 1, 40], \text{Cos}[\text{Pi}\,x/2]\}, \{x, 0, 1\}, \text{PlotRange} \rightarrow \{-6, 6\}]$$

```
(* one period [0, 4]    *)
```
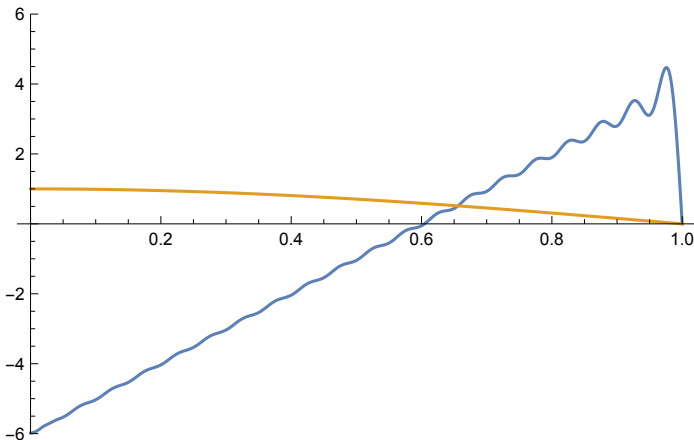
```
Plot[{v[x, 0, 1, 40], Cos[Pi x/2]}, {x, 0, 4}, PlotRange → {-6, 6}]
```
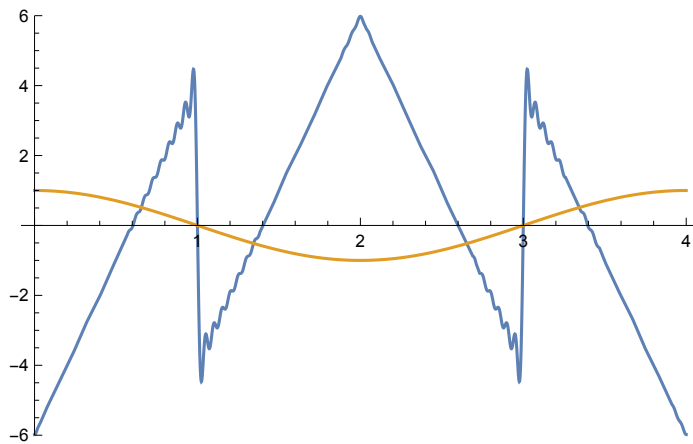


```
(* Very rapid smoothing t = {0.001, 0.01, 0.05}  *)

Plot[{v[x, 0.001, 1, 40], v[x, 0.01, 1, 40], v[x, 0.05, 1, 40]},
 {x, 0, 4}, PlotRange → {-6, 6}]
```