CONCORDIA UNIVERSITY **Department of Mathematics and Statistics**

Course	Number	Section(s)
Mathematics	**MAST 218**	**A, B**

Examination	Date	Time	Pages
Final	**December 2014**	**3 hours**	**2**

Instructors	Course Examiner
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Special Instructions: Calculators permitted. Lined paper booklets.

READ THE QUESTIONS CAREFULLY !!!

SHOW ALL WORK !!! JUSTIFY ALL STEPS !!!

GOOD LUCK !!!

MARKS: marks for each problem are shown in front of the problems.

↓MARKS

10 **Problem 1 :** Consider the plane curve defined by the parametric equations: $x = \sin^2 t$, $y = \cos t$, $0 \leq t \leq 2\pi$.

(a). Find d^2y/dx^2 in terms of t.

(b). Find the values of t at which the plane curve is concave upward.

10 **Problem 2 :** Consider the curves γ_1 and γ_2 defined by polar equations:

$$\gamma_1 : \quad r = 1 - \cos\theta, \qquad \gamma_2 : \quad r = 1 + \cos\theta.$$

(a). Sketch the polar curves γ_1 and γ_2.

(b). Find the area A of the region which lies inside γ_1 and outside γ_2.

10 **Problem 3 :** Consider the function

$$f(x) = ln\left(\frac{1+x}{1-x}\right)$$

(a). Find the Taylor series of $f(x)$ at $x = 0$.

(b). Find the interval of convergence for the Taylor series obtained in (a).

10 **Problem 4 :** (a). Find an equation of the plane passing through the point $A\,(1,2,3)$ and the line $(3t,2t,t)$, $-\infty < t < +\infty$.

(b). Find an equation of the line passing through the point $(2,2,0)$ and perpendicular to the plane obtained in (a).

(c). Find the distance from the point $(2,2,0)$ to the plane obtained in (a).

10 **Problem 5 :** Consider the space curve $\mathbf{r}(t) = \langle 3t\cos t,\, 4t,\, 3t\sin t\rangle$.

(a). Find an equation of the tangent line to the curve at $\mathbf{r}(1)$.

(b). Find the length of the curve for $t \in [0,4]$. You can use a formula

$$\int \sqrt{1+u^2}du = \frac{u}{2}\sqrt{1+u^2} + \frac{1}{2}\ln(u+\sqrt{1+u^2}) + C\,.$$

10 **Problem 6 :** Consider the space curve $\mathbf{r}(t) = \langle t,\, 2t,\, t^2\rangle$.

(a). Find the unit tangent vector $\mathbf{T}(1)$ and the principal normal vector $\mathbf{N}(1)$ of the curve at $t = 1$.

(b). **Use the Chain Rule** to find the partial derivatives $\frac{\partial u}{\partial s}$ and $\frac{\partial u}{\partial t}$, where

$$u = xe^{yz^2}, \quad x = \ln(st), \quad y = t^3, \quad z = s^2 + t^2.$$

10 **Problem 7 :** Find the limit, if it exists, or show that the limit does not exist:

$$(a). \quad \lim_{(x,y)\to(0,0)} \frac{3xy^2}{x^2+y^2}\,, \qquad\qquad (b). \quad \lim_{(x,y)\to(0,0)} \frac{\sin(x^2y^2)}{x^4+y^4}.$$

10 **Problem 8 :** Consider the function $f(x,y) = (y^2+x^2)e^{x^2-y^2}$

(a). Find all critical points of $f(x,y)$.

(b). Classify those critical points obtained in (a) as points of local minimum, local maximum, or saddle points.

10 **Problem 9 :** For the function $f(x,y) = 3-2x+y+xy$, find the absolute maximum and minimum values of $f(x,y)$ in the region enclosed by the curves $y = x^2$ and $y = 4$.

10 **Problem 10 :** Use the **Lagrange Multipliers** to find the maximum and minimum values of $f(x,y) = xy^2$ subject to the constraint: $4x^2 + 9y^2 = 36$.