Midterm Test, MAST 218, Fall 2014

All problems have equal value. Each is worth 6 marks.

Show all your steps. Write the solutions in the examination booklet only.

Problem 1:   The curve is given parametrically:

$$x(t) = e^{2t} \cos(2t) \quad , \quad y(t) = e^{2t} \sin(2t) \; , \quad t \in [0, \pi/2] \; .$$

(1). Find the length L of the curve.

(2). At which points the tangent to the curve is vertical? horizontal?

Problem 2:   For the curve given by:

$$r(\theta) = 3 + 3 \sin \theta \quad , \quad 0 \le \theta \le 2\pi \; ,$$

(1). Give a rough sketch of the curve.

(2). Calculate the area enclosed by the curve.

Problem 3:   For the curve:

$$r = \frac{8}{2 + 8 \sin \theta}.$$

(1). Find the eccentricity and the directrix.

(2). Sketch the curve.

Problem 4:   (a). Find an equation of the plane passing through the three points: $A\,(0, 3, 2)$, $B\,(-2, 1, 0)$, and $C\,(3, 2, 4)$.

(b). Find an equation of the line passing through the point $(1, -2, 0)$ and perpendicular to the plane obtained in (a).

(c). Find the distance from the point $(1, -2, 0)$ to the plane obtained in (a).

Problem 5:   (1). Find a Taylor series of

$$f(x) = \ln(1 + 7x^2) \quad \text{at} \quad a = 0 \; .$$

(2). Find the radius R of convergence for the Taylor series obtained in (1).

GOOD LUCK !!!