CONCORDIA UNIVERSITY **Department of Mathematics and Statistics**

Course	Number	Section(s)
Mathematics	**MAST 218**	**A, B**

Examination	Date	Time	Pages
Final	**December 2012**	**3 hours**	**2**

Instructors	Course Examiner
John Harnad, Ewa Duma	**Ewa Duma**

Instructions: Only approved calculators permitted. Answer all eight numbered questions. The value for each part is indicated in square brackets in the margin (out of a possible total of 100). Show all your steps. Write the complete solution on the right hand pages of your examination booklet only.

MARKS: marks for each problem are shown in front of the problems.

↓MARKS

12 **Problem 1 :** A curve is defined y the parametric equations

$$x = \int_1^t \frac{\cos u}{u} du, \quad y = \int_1^y \frac{\sin u}{u} du,$$

Find the length of the arc of the curve from the origin to the nearest point where there is a vertical tangent line.

12 **Problem 2 :** Find the area of the region that lies inside the curve $r + \cos(2\theta)$ but outside the curve $r = 2 + \sin\theta$.

12 **Problem 3 :** Find an equation of the plane that passes through the line of intersection of the planes $x - z = 1$ and $y + 2z = 3$ and is perpendicular to the plane $x + y - 2z = 1$.

14 **Problem 4 :**

(a) Find an equation of the plane that passes through the points $A(2, 1, 1)$, $B(-1, -1, 10)$ and $C(1, 3, -4)$.

(b) Find symmetric equations for the line through B that is perpendicular to the plane in part (a).

(c) A second plane passes through $(2, 0, 4)$ and has normal vector $(2, -4, 03)$. Find parametric equations for the line of intersection of the two planes.

12 **Problem 5 :**

Reparametrize the curve

$$\mathbf{r}(t) = e^t \ \mathbf{i} + e^t \sin t \ \mathbf{j} + e^t \cos t \ \mathbf{k}$$

with respect to arc length measured from the point $(1, 0, 1)$ in the direction of increasing t.

12 **Problem 6 :**

Find equations of (a) the tangent plane and (b) the normal line to the surface

$$z = e^x \cos y$$

at the point $(0, 0, 1)$.

12 **Problem 7 :**

Find the directional derivative of

$$f(x, y) = x^2 e^{-y}$$

at the point $(-2, 0)$ in the direction towards the point $(2, -3)$.

14 **Problem 8 :**

Use **Lagrange Multipliers** to find the maximum and minimum values of

$$f(x, y, z) = x^2 + 2y^2 + 3z^2$$

subject to the constraints

$$x + y + z = 1, \quad x - y + 2z = 2.$$