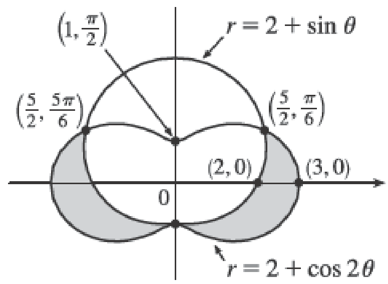
36. $A = 2\int_{-\pi/2}^{\pi/6} \frac{1}{2}\left[(2+\cos 2\theta)^2 - (2+\sin\theta)^2\right] d\theta$

$= \int_{-\pi/2}^{\pi/6} \left[4\cos 2\theta + \cos^2 2\theta - 4\sin\theta - \sin^2\theta\right] d\theta$

$= \left[2\sin 2\theta + \frac{1}{2}\theta + \frac{1}{8}\sin 4\theta + 4\cos\theta - \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]_{-\pi/2}^{\pi/6}$

$= \frac{51}{16}\sqrt{3}$



1. $x = \displaystyle\int_1^t \frac{\cos u}{u}\,du$, $y = \displaystyle\int_1^t \frac{\sin u}{u}\,du$, so by FTC1, we have $\dfrac{dx}{dt} = \dfrac{\cos t}{t}$ and $\dfrac{dy}{dt} = \dfrac{\sin t}{t}$. Vertical tangent lines occur when $\dfrac{dx}{dt} = 0 \quad\Leftrightarrow\quad \cos t = 0$. The parameter value corresponding to $(x, y) = (0, 0)$ is $t = 1$, so the nearest vertical tangent occurs when $t = \frac{\pi}{2}$. Therefore, the arc length between these points is

$$L = \int_1^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_1^{\pi/2} \sqrt{\frac{\cos^2 t}{t^2} + \frac{\sin^2 t}{t^2}}\,dt = \int_1^{\pi/2} \frac{dt}{t} = \left[\ln t\right]_1^{\pi/2} = \ln\frac{\pi}{2}$$

25. $\mathbf{n_1} = \langle 1, 0, -1\rangle$ and $\mathbf{n_2} = \langle 0, 1, 2\rangle$. Setting $z = 0$, it is easy to see that $(1, 3, 0)$ is a point on the line of intersection of $x - z = 1$ and $y + 2z = 3$. The direction of this line is $\mathbf{v_1} = \mathbf{n_1} \times \mathbf{n_2} = \langle 1, -2, 1\rangle$. A second vector parallel to the desired plane is $\mathbf{v_2} = \langle 1, 1, -2\rangle$, since it is perpendicular to $x + y - 2z = 1$. Therefore, the normal of the plane in question is $\mathbf{n} = \mathbf{v_1} \times \mathbf{v_2} = \langle 4 - 1, 1 + 2, 1 + 2\rangle = 3\langle 1, 1, 1\rangle$. Taking $(x_0, y_0, z_0) = (1, 3, 0)$, the equation we are looking for is $(x - 1) + (y - 3) + z = 0 \quad\Leftrightarrow\quad x + y + z = 4$.

26.

(a) The vectors $\overrightarrow{AB} = \langle -1-2, -1-1, 10-1 \rangle = \langle -3, -2, 9 \rangle$ and $\overrightarrow{AC} = \langle 1-2, 3-1, -4-1 \rangle = \langle -1, 2, -5 \rangle$ lie in the plane, so $\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \langle -3, -2, 9 \rangle \times \langle -1, 2, -5 \rangle = \langle -8, -24, -8 \rangle$ or equivalently $\langle 1, 3, 1 \rangle$ is a normal vector to the plane. The point $A(2, 1, 1)$ lies on the plane so an equation of the plane is $1(x-2) + 3(y-1) + 1(z-1) = 0$ or $x + 3y + z = 6$.

(b) The line is perpendicular to the plane so it is parallel to a normal vector for the plane, namely $\langle 1, 3, 1 \rangle$. If the line passes through $B(-1, -1, 10)$ then symmetric equations are $\dfrac{x-(-1)}{1} = \dfrac{y-(-1)}{3} = \dfrac{z-10}{1}$ or $x + 1 = \dfrac{y+1}{3} = z - 10$.

(c) Normal vectors for the two planes are $\mathbf{n}_1 = \langle 1, 3, 1 \rangle$ and $\mathbf{n}_2 = \langle 2, -4, -3 \rangle$. The angle θ between the planes is given by

$$\cos\theta = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1|\ |\mathbf{n}_2|} = \frac{\langle 1, 3, 1 \rangle \cdot \langle 2, -4, -3 \rangle}{\sqrt{1^2 + 3^2 + 1^2}\,\sqrt{2^2 + (-4)^2 + (-3)^2}} = \frac{2 - 12 - 3}{\sqrt{11}\,\sqrt{29}} = -\frac{13}{\sqrt{319}}$$

Thus $\theta = \cos^{-1}\left(-\dfrac{13}{\sqrt{319}}\right) \approx 137^\circ$ or $180^\circ - 137^\circ = 43^\circ$.

(d) From part (c), the point $(2, 0, 4)$ lies on the second plane, but notice that the point also satisfies the equation of the first plane, so the point lies on the line of intersection of the planes. A vector $\mathbf{v}$ in the direction of this intersecting line is perpendicular to the normal vectors of both planes, so take $\mathbf{v} = \mathbf{n}_1 \times \mathbf{n}_2 = \langle 1, 3, 1 \rangle \times \langle 2, -4, -3 \rangle = \langle -5, 5, -10 \rangle$ or equivalently we can take $\mathbf{v} = \langle 1, -1, 2 \rangle$. Parametric equations for the line are $x = 2 + t$, $y = -t$, $z = 4 + 2t$.

10. The parametric value corresponding to the point $(1, 0, 1)$ is $t = 0$.

$\mathbf{r}'(t) = e^t\,\mathbf{i} + e^t(\cos t + \sin t)\,\mathbf{j} + e^t(\cos t - \sin t)\,\mathbf{k} \quad \Rightarrow \quad |\mathbf{r}'(t)| = e^t\sqrt{1 + (\cos t + \sin t)^2 + (\cos t - \sin t)^2} = \sqrt{3}\,e^t$

and $s(t) = \int_0^t e^u \sqrt{3}\ du = \sqrt{3}(e^t - 1) \quad \Rightarrow \quad t = \ln\!\left(1 + \frac{1}{\sqrt{3}}s\right)$.

Therefore, $\mathbf{r}(t(s)) = \left(1 + \frac{1}{\sqrt{3}}s\right)\mathbf{i} + \left(1 + \frac{1}{\sqrt{3}}s\right)\sin\ln\!\left(1 + \frac{1}{\sqrt{3}}s\right)\mathbf{j} + \left(1 + \frac{1}{\sqrt{3}}s\right)\cos\ln\!\left(1 + \frac{1}{\sqrt{3}}s\right)\mathbf{k}$.

14. $g(u, v) = \dfrac{u + 2v}{u^2 + v^2} \quad \Rightarrow \quad g_u = \dfrac{(u^2 + v^2)(1) - (u + 2v)(2u)}{(u^2 + v^2)^2} = \dfrac{v^2 - u^2 - 4uv}{(u^2 + v^2)^2}$,

$g_v = \dfrac{(u^2 + v^2)(2) - (u + 2v)(2v)}{(u^2 + v^2)^2} = \dfrac{2u^2 - 2v^2 - 2uv}{(u^2 + v^2)^2}$

45. $f(x, y) = x^2 e^{-y} \quad \Rightarrow \quad \nabla f = \langle 2xe^{-y}, -x^2 e^{-y} \rangle$, $\nabla f(-2, 0) = \langle -4, -4 \rangle$. The direction is given by $\langle 4, -3 \rangle$, so

$\mathbf{u} = \frac{1}{\sqrt{4^2 + (-3)^2}}\langle 4, -3 \rangle = \frac{1}{5}\langle 4, -3 \rangle$ and $D_{\mathbf{u}}\,f(-2, 0) = \nabla f(-2, 0) \cdot \mathbf{u} = \langle -4, -4 \rangle \cdot \frac{1}{5}\langle 4, -3 \rangle = \frac{1}{5}(-16 + 12) = -\frac{4}{5}$.

62.
$f(x,y,z) = x^2 + 2y^2 + 3z^2,\ g(x,y,z) = x + y + z = 1,\ h(x,y,z) = x - y + 2z = 2 \quad \Rightarrow$

$\nabla f = \langle 2x, 4y, 6z \rangle = \lambda \nabla g + \mu \nabla h = \langle \lambda + \mu, \lambda - \mu, \lambda + 2\mu \rangle$ and $2x = \lambda + \mu$ **(1)**, $\quad 4y = \lambda - \mu$ **(2)**, $\quad 6z = \lambda + 2\mu$ **(3)**,

$x + y + z = 1$ **(4)**, $\quad x - y + 2z = 2$ **(5)**. Then six times **(1)** plus three times **(2)** plus two times **(3)** implies

$12(x + y + z) = 11\lambda + 7\mu$, so **(4)** gives $11\lambda + 7\mu = 12$. Also six times **(1)** minus three times **(2)** plus four times **(3)** implies

$12(x - y + 2z) = 7\lambda + 17\mu$, so **(5)** gives $7\lambda + 17\mu = 24$. Solving $11\lambda + 7\mu = 12$, $7\lambda + 17\mu = 24$ simultaneously gives

$\lambda = \frac{6}{23}$, $\mu = \frac{30}{23}$. Substituting into **(1)**, **(2)**, and **(3)** implies $x = \frac{18}{23}$, $y = -\frac{6}{23}$, $z = \frac{11}{23}$ giving only one point. Then

$f\left(\frac{18}{23}, -\frac{6}{23}, \frac{11}{23}\right) = \frac{33}{23}$. Now since $(0, 0, 1)$ satisfies both constraints and $f(0, 0, 1) = 3 > \frac{33}{23}$, $f\left(\frac{18}{23}, -\frac{6}{23}, \frac{11}{23}\right) = \frac{33}{23}$ is an absolute minimum, and there is no absolute maximum.