Mast 218 (B) Test Solution Notes October 2015

Professor: Richard Hall

1. (a) We use a distinct notation for an arbitrary point $\mathbf{p}$ in the plane. Thus the equation to the plane is given by $(\mathbf{p}-\mathbf{r}(1))\cdot\mathbf{r}'(1)=0$, where $\mathbf{r}(1)=(1,-1,1)$ and $\mathbf{r}'(1)=(1,-2,3)$.

   (b) The curvature of the curve at $t=1$ is given by $\kappa=|\mathbf{r}'(1)\times\mathbf{r}''(1)|/|\mathbf{r}'(1)|^3=\frac{2\sqrt{19}}{14\sqrt{14}}$.
   Thus $R=\frac{1}{\kappa}=7\frac{\sqrt{14}}{\sqrt{19}}\approx 6.009$

2. (a) Let the three sides of the block from the origin be $\{\mathbf{a},\mathbf{b},\mathbf{c}\}=\{(1,0,0),(0,2,0),(0,0,3)\}$, then the $\{1-2,\,2-3\}$ diagonals are parallel respectively to the vectors $\{\mathbf{a}+\mathbf{b},\,\mathbf{b}+\mathbf{c}\}$. The required angle is therefore given by
   $\cos^{-1}\left(\frac{(\mathbf{a}+\mathbf{b})\cdot(\mathbf{b}+\mathbf{c})}{|(\mathbf{a}+\mathbf{b})||(\mathbf{b}+\mathbf{c})|}\right)=\cos^{-1}(\frac{4}{\sqrt{5}\sqrt{13}})\approx\cos^{-1}(0.496139)=1.05153=60.248^0$.

   (b) The area of the parallelogram is $|(\mathbf{a}+\mathbf{b}+\mathbf{c})\times(\mathbf{b}+\mathbf{c})|=\sqrt{13}$.

3. (a) A graph of the curve is:

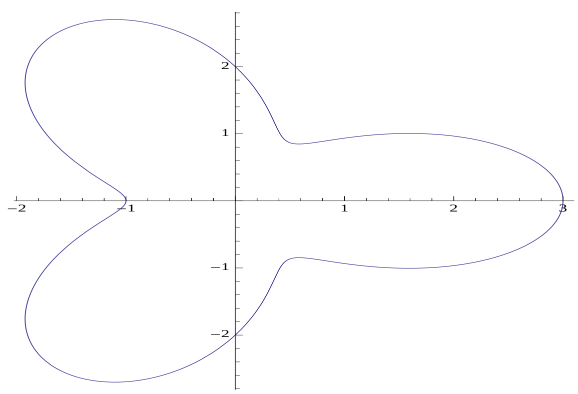

Figure 1: for Q3(a)

   (b) The parametric form of the curve is given by $x=r(\theta)\cos(\theta)$, $\quad y=r(\theta)\sin(\theta)$. Thus the slope becomes

$$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{\cos(\theta)r(\theta)+\sin(\theta)r'(\theta)}{-\sin(\theta)r(\theta)+\cos(\theta)r'(\theta)},$$

   where $r'(\theta)=-3\sin(3\theta)$. At $\theta=\pi/2$ we have $(x,y)=(0,2)$, $r(\theta)=2$, $r'(\theta)=3$, and the slope is given by $dy/dx=-3/2$.

4. (a) The curve is an ellipse with semi x-axis $a = \frac{1}{2}(1 - -9) = 5$, and $b = 3$. Since $1 - e^2 = (b/a)^2$, the eccentricity $e = 4/5$. The centre of the ellipse is at $x = -4$ in the coordinates of the graph: we can think of this as a shift to the left by 4 units of a centrally placed ellipse. The polar equation in standard form is $r = de/(1 + e\cos(\theta))$. Thus we know that $r(0) = de/(1 + e) = 1$; hence $d = 9/4$.

   (b) The polar expression of this curve is:

$$r(\theta) = \frac{9/5}{1 + (4/5)\cos(\theta)} = \frac{9}{5 + 4\cos(\theta)}.$$

   In cartesians we have

$$\frac{(x+4)^2}{5^2} + \frac{y^2}{3^2} = 1.$$

   The coordinate x becomes $(x + 4)$ since a corresponding centrally-placed ellipse must be moved to the left.