Mast 218 Practice Test Solution Notes 2015

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1. (a) Two sides of the triangle in PL are $\mathbf{a} = (B - A)$ and $\mathbf{b} = (C - A)$. Thus a normal to the plane is $\mathbf{a} \times \mathbf{b} = (-8, 12, -10)$ and we set $\mathbf{n} = (-4, 6, -5)$. Meanwhile, a point in the plane is $\mathbf{c} = A = (1, 0, 1)$. Thus the equation of the plane PL is $(\mathbf{r} - \mathbf{c}) \cdot \mathbf{n} = 0$.

   (b) Hence, a line through the origin perpendicular to the plane PL is given by $\mathbf{r}(t) = t\mathbf{n}$.

2. (a) The graph is a cardoid:

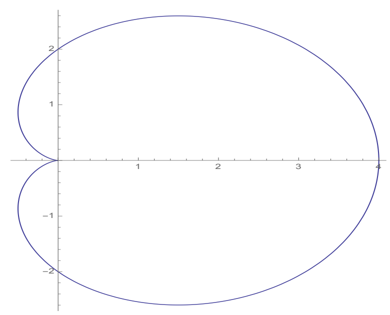

Figure 1: for Q2(a)

   (b) The area $A = \int_0^{2\pi} \frac{1}{2} r^2(\theta) d\theta = \int_0^{2\pi} \frac{1}{2}(2 + 2\cos(\theta))^2 d\theta$.
   We substitute $\cos^2\theta = (\cos 2\theta + 1)/2$, integrate, and find $A = 6\pi$.

3. (a) $r = 6/(2 + 6\sin\theta) = 3/(1 + 3\sin\theta) = de/(1 + e\sin\theta)$. Hence $e = 3$ and the directrix is at $y = d = 1$. This is found by starting in standard form, with $\cos\theta$, and rotating anticlockwise by $\pi/2$. The apexes are at $r(\pi/2) = 3/4$ and $r(3\pi/2) = -3/2$, that is to say $y = 3/4$ and $y = 3/2$.

   (b) The curve is a hyperbola with symmetry about the y-axis.

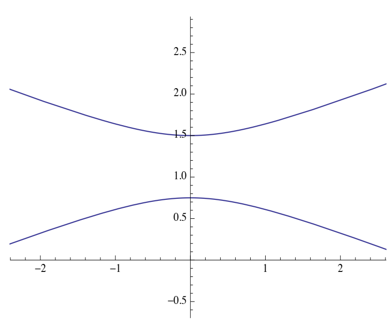

Figure 2: for Q3(b)

4. (a) $L = \int_0^{2\pi} |\mathbf{r}'(t)| dt = \int_0^{2\pi} e^{-t} 3^{\frac{1}{2}} dt = \left[-e^{-t} 3^{\frac{1}{2}}\right]_0^{2\pi} = \left(1 - e^{-2\pi}\right) 3^{\frac{1}{2}}$.

   (b) If the local tangent vector $\mathbf{r}'(t)$ were parallel to z, at t_1, the x and y components would both have to vanish at t_1. But this is impossible for it implies $(\cos t_1 - t_1 \sin t_1 = t_1 \cos t_1 + \sin t_1) \Rightarrow t_1^2 = -1$.