FINAL MATH 265

Evaluation out of 100. Calculators are NOT allowed.

1. (8) Find the volume of the solid enclosed by the parabolic cylinders $y = 1 - x^2$, $y = x^2 - 1$ and the planes $x + y + z = 2$, $2x + 2y - z + 10 = 0$.

2. (8) Use a double integral to find the area of the region within both of the circles $r = \sin\theta$ and $r = \cos\theta$ ((r, θ) are polar coordinates).

3. (8) The joint density function for a pair of random variables X and Y is

$$f(x, y) = Cx(1 + y) \text{ if } 0 \le x \le 1\ ,\ \ 0 \le y \le 2$$

and $f(x, y) = 0$ otherwise. Find a) the value of constant C; b) $P(X \le 1, Y \le 1)$; c) $P(X+Y \le 1)$

4. (8) Find the volume enclosed by the torus $\rho = \sin\varphi$ ((ρ, θ, φ) are spherical coordinates).

5. (8) Find the volume and centroid of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

6. (8) Evaluate

$$\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2 + y^2 + z^2}\, dz\, dx\, dy$$

7. (8) Evaluate $\iint_R \cos\left\{\frac{y-x}{y+x}\right\} dA$ using an appropriate change of variables, where R is the trapezoidal region with vertices (1, 0), (2, 0), (0, 2) and (0, 1).

8. (8) Find the work done by he force field $\mathbf{F}(x, y, z) = (y + z)\mathbf{i} + (x + z)\mathbf{j} + (x + y)\mathbf{k}$ on a particle that moves along the line segment from $(1, 0, 0)$ to $(3, 4, 2)$.

9. (8) Find the area of the helicoid $\mathbf{r}(u, v) = u\cos v\mathbf{i} + u\sin v\mathbf{j} + v\mathbf{k}$, $0 \le u \le 1$, $0 \le v \le \pi$

10. (8) Evaluate the surface integral $\iint_S \mathbf{F} d\mathbf{S}$ if $F = x\mathbf{i} + y\mathbf{j} + 2z\mathbf{k}$ and S is the surface of the solid bounded by hemisphere $x^2 + y^2 + z^2 \le a^2$, $z \ge 0$ and the plane $z = 0$ with outward orientation.

11. (8) Evaluate $\iint_S \operatorname{curl}\mathbf{F}\ d\mathbf{S}$ where

$$\mathbf{F}(x, y, z) = x^2yz\mathbf{i} + yz^2\mathbf{j} + z^3e^{xy}\mathbf{k}$$

and S is the part of the sphere $x^2 + y^2 + z^2 = 5$ that lies above the plane $z = 1$ and S is oriented upwards.

12. (8) Solve the initial value problem

$$y'' + y' - 2y = x + \sin 2x\ ,\ \ y(0) = 1\ ,\ \ y'(0) = 0\ .$$

13. (6) A force of $400N$ stretches a spring 2 meters. A mass of 50 kilogramms is attached to the end of the spring and released from the equilibrum position with an upward velocity of $10m/s$. Find position of the mass as function of time.