Math 265 Secs A & AA Final Exam April 2006

Professors: *Richard Hall, Dima Korotkin*
Instructions: *Please answer all 13 questions, which carry equal marks. Calculators are not permitted. [Lined Booklets]*

(1) Find the volume of the solid enclosed by the parabolic cylinders $y = 1 - x^2$, $y = x^2 - 1$ and the planes $x + y + z = 2$, $2x + 2y - z + 10 = 0$.

(2) Use a double integral to find the area of the region within both of the circles $r = \sin\theta$ and $r = \cos\theta$ ((r, θ) are polar coordinates).

(3) The joint density function for a pair of random variables X and Y is

$$f(x, y) = Cx(1 + y) \text{ if } 0 \le x \le 1, \ 0 \le y \le 2$$

and $f(x, y) = 0$ otherwise. Find a) the value of constant C; b) $P(X \le 1, Y \le 1)$; c) $P(X + Y \le 1)$

(4) Find the volume enclosed by the torus $\rho = \sin\varphi$ ((ρ, θ, φ) are spherical coordinates).

(5) Find the volume and centroid of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

(6) Evaluate

$$\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2 + y^2 + z^2} dz\, dx\, dy$$

(7) Evaluate $\int\int_R \cos\left\{\frac{y-x}{y+x}\right\} dA$ using an appropriate change of variables, where R is the trapezoidal region with vertices $(1, 0)$, $(2, 0)$, $(0, 2)$ and $(0, 1)$.

(8) Find the work done by he force field $F(x, y, z) = (y + z)i + (x + z)j + (x + y)k$ on a particle that moves along the line segment from $(1, 0, 0)$ to $(3, 4, 2)$.

(9) Find the area of the helicoid $r(u, v) = u\cos v i + u\sin v j + vk$, $0 \le u \le 1$, $0 \le v \le \pi$

(10) Evaluate the surface integral $\int\int_S F dS$ if $F = xi + yj + 2zk$ and S is the surface of the solid bounded by hemisphere $x^2 + y^2 + z^2 \le a^2$, $z \ge 0$ and the plane $z = 0$ with outward orientation.

(11) Evaluate $\int\int_S \text{curl}\, F\, dS$ where

$$F(x, y, z) = x^2 yz i + yz^2 j + z^3 e^{xy} k$$

and S is the part of the sphere $x^2 + y^2 + z^2 = 5$ that lies above the plane $z = 1$ and S is oriented upwards.

(12) Solve the initial value problem

$$y'' + y' - 2y = x + \sin 2x \ , \ \ y(0) = 1 \ , \ \ y'(0) = 0 \ .$$

(13) A force of $400N$ stretches a spring 2 meters. A mass of 50 kilogramms is attached to the end of the spring and released from the equilibrum position with an upward velocity of $10m/s$. Find position of the mass as function of time.