Concordia University Department of Mathematics and Statistics

Mast 219 Sec. AA Midterm Test 3rd March 2016 (Duration $1\frac{1}{4}$ hrs.)

Professor: Richard Hall

Instructions: Please answer all 3 questions, which carry equal marks.
Show your work clearly. Note that only approved calculators are allowed.

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1. (a) sketch the region D that is in the first quadrant and lies between the two circles $x^2 + y^2 = 4$ and $x^2 + y^2 = x$.

   (b) Find the value of the integral

$$I = \iint_D x dA, \tag{1}$$

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2. (a) By means of a double integral, find the surface area $S(1)$ of that part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the plane $z = 1$.

   (b) By examining your solution to (a), or otherwise, derive a formula for $S(h)$, where the plane is now given by $z = h$, and h is a constant satisfying $0 \leq h \leq 2$.

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3. Find the position of the centroid of a solid hemisphere of radius a whose density is given by Kz, where K is a positive constant and z is the perpendicular distance from an arbitrary point of the solid to the circular base of the solid.
