

Math 265 Sec AA Midterm Test 25 Feb 2002

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Instructions: Please answer questions (1) and (2) and one of questions (3) and (4). Explain your work clearly. Calculators are permitted.

1. Find the integral I of the function $f(x, y) = ye^x$ over the triangular region of the plane with vertices $\{(0, 0), (6, 0), (2, 4)\}$.
2. Find the volume inside the cylinder $x^2 + 4y^2 = 4$, above the xy -plane, and beneath the plane $z = 2 + x$.
3. Consider the *probability density function* $\rho(x, y) = Ce^{-\alpha(x^2+y^2)}$ defined on the xy -plane $D = \mathbb{R}^2$, where α is a positive constant.
 - (a) Find the value of C so that the total probability $\iint_D \rho(x, y) dA = 1$.
 - (b) Find the mean value \bar{f} of $f(x, y) = 2x^3 + 3y^2$ with respect to ρ . [HINT: symmetry]
4.
 - (a) State the *perpendicular-axes theorem* and the *parallel axes theorem* concerning moments of inertia.
 - (b) Find the moment of inertia I_X of a hollow sphere of radius a and constant area density σ about an axis X which passes through the centre of the sphere.
 - (c) Find the moment of inertia I_Y of a hollow sphere of radius a and constant area density σ about an axis Y which is tangent to the sphere.