

Math 265 Midterm Test 15 February 2006

Professor: Richard Hall

Instructions: Please answer all 5 questions.

Explain your work clearly. Calculators are not permitted.

1. Evaluate the integral $I = \iint_D (x^2(1 + \sin x) + x^4 \tan y + x^2) dA$,
where $D = \{(x, y) | x^2 + y^2 \leq 4\}$.
2. Find the centroid of the region bounded below by $z = 0$ and above by $z = 1 - x^2 - y^2$.
3. If $\rho(x, y) = Ce^{-4x^2 - 9y^2}$ is a probability density on \mathbb{R}^2 , find (i) C and (ii) the mean \bar{f} of $f(x, y) = (x - y)^2$.
4. Find the moments of inertia of a thin circular disc of radius a and mass M about (i) an axis perpendicular to the disc and a distance a from the centre, and (ii) an axis in the plane of the disc that is $a/3$ from a parallel axis through the centre of the disc. [HINT: recall the parallel-axes and perpendicular-axes theorems shown below.]
5. Find the value of the integral $I = \iint_D y^2 dA$, where D is the region bounded by the four curves $xy = 1$, $xy = 2$, $xy^2 = 2$, $xy^2 = 4$.

Parallel axes theorem If I_X is the moment of inertia about an axis through the centre of mass, then the moment of inertia about a parallel axis X' distance d from X is given by $I_{X'} = I_X + Md^2$.

Perpendicular axes theorem If $\{X, Y, Z\}$ are perpendicular axes through the centre of area of a flat plate, and $\{X, Y\}$ are in the plane of the plate, then $I_Z = I_X + I_Y$.
