Math 683 Assignment 3

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Instructions:	*Please use file names and identifiers exactly as requested.*
Due Date:	*14 November 2000*

(3.1) Construct the 'toolbox' function `double gauss(fptype1 f, double a, double b, int m = 1)` for 10-point Gauss-Legendre integration and overload it for 'fun' objects. If $m > 1$, the interval $[a, b]$ is divided into m sub-intervals, the routine is applied to each, and the results are summed. Test `gauss` first of all by applying it to the function $f(x) = 1 - 2x + 5x^4 - 8x^7 - 13x^{12} + 15x^{14} + 20x^{19}$ on $[0, 1]$. Now consider definite integrals of the functions

$$f_1(x) = |x|, \quad f_2(x) = 2|x|^3, \quad \text{and} \quad f_3(x) = [1 - x^2]^{\frac{1}{2}} \quad \text{on} \quad [-1, 1].$$

Find the percentage errors if $m = 1$. These results are a warning: each method always has its limitations. Try to find in each case a sub-division policy that greatly reduces the error: this might be called 'adaptive quadrature by hand'. It would be interesting to find a Lagrange polynomial approximation of degree 6, say, to $f_2(x)$ to demonstrate how surprisingly *unlike* a polynomial the function $f_2 \in C_2(\mathbb{R})$ is.

(3.2) Construct the inverse function `double binv(fptype1 f, double y, double x1, double x2, double ytol = 1e-6)` and also the 'fun' version (overloading the identifier *binv*) `double binv(fun *f, double y, double x1, double x2, double ytol = 1e-6)` Define these two functions in a file called binv.h and write a program binvtest.cpp which demonstrates their use.

(3.3) Consider the class `dif` discussed in the lectures which constructs from a 'fun' object corresponding to the function $f(x)$ a new 'fun' object corresponding (approximately) to $f'(x)$. Now construct the classes (a) `dif1(fun *f, double h = 1e-3)` which is an improvement on `dif` because the error is now $\sim h^4$, (b) `fgauss(fun *f, double a = 0)` corresponding to the integral $\int_a^x f(t)dt$, and (c) the `fbinv(fun *f, double a, double b, double ytol = 1e-6)` corresponding to $f^{-1}(x)$. For fgauss use Gauss-Legendre integration with $m = 1$; for fbinv use binv. Now write a short graphics program that demonstrates dif1, fgauss, and fbinv.