Classical Optimization Math 618K

Professor: Richard Hall Office: LB 541-3 Telephone: 848-2424 loc 3221 or 3250;
Email: richard.hall@concordia.ca
Web site: www.mathstat.concordia.ca/faculty/rhall/

Texts: A reading list will be provided. An excellent book to own is *Calculus of Variations,* by I. M. Gelfand and V. S. Fomin (Prentice-Hall, 1963; Dover 2000 [ISBN 0486414485]); it sells for $< \$10$, for example, at Amazon: http://www.amazon.com/Calculus-Variations-Dover-Books-Mathematics/dp/0486414485.

Background: It will be assumed that students have studied advanced calculus and differential equations. Students will be encouraged to explore problems and their solutions with the aid of a computer and suitable software such as *Maple* or in-house programs available on my web page.

Topics: What is the path of a ray of light if the refractive index varies? If a surface has a given boundary, what shape would minimize its surface area? (A soap film 'naturally' solves this problem.) In the design of a boat, what shape would minimize the viscous drag? Problems such as these may be formulated and solved with the aid of the 'calculus of variations' (classical optimization). We study the general problem of finding a curve or surface that minimizes a 'cost function'. A best curve or surface must be locally optimal, it must satisfy Euler's differential equation. In effect, this course applies calculus and differential equations to geometrical and physical problems.

Evaluation: There will be assignments (50%), and individual projects (50%). A project is essentially an expository mathematical essay. Although the subject of the course is mathematics rather than pedagogy, appropriate ancillary material in the form, for example, of computer programs, photographs, or video clips, would be welcome as part of a project. Students will have an opportunity during the course of presenting a seminar on their projects.

Classical Optimization Reading List

J. C. Clegg *Calculus of Variations* (Oliver and Boyd, Edinburgh, 1968). A good elementary book with many simple examples and problems.

I. M. Gelfand and S. V. Fomin *Calculus of Variations* (Prentice-Hall, New Jersey, 1963). An excellent exposition in the classical style, including some discussion of mechanics and optimal control.

L. Pars *An Introduction to the Calculus of Variations* (Heinemann, London, 1962). A delightful classic with many solved problems.

J. L. Troutman *Variational Calculus with Elementary Convexity* (Springer-Verlag, New York, 1983). A modern undergraduate text. An excellent presentation of the mathematical ideas behind classical optimization.

M. J. Sewell *Maximum and Minimum Principles* (Cambridge U.P., Cambridge, 1987). A modern undergraduate text with a very broad range of applications treated in detail.

H. Sagan *Introduction to the Calculus of Variations* (McGraw-Hill, New York, 1969). This great book is unforunately out of print. The style is modern and very precise with many details worked out. See also his excellent book on advanced calculus.

C. Carathodory *Calculus of Variations and Partial Differential Equations of the First Order* (Chelsea, New York, 1982). A re-issue of a grand old classic. If you like Sommerfeld for mechanics, then this is the corresponding grand master to look at for calculus of variations. It's not easy. The style is more concrete than many contemporary works; this feature is sometimes welcome and sometimes not.

K. Rektorys *Variational Methods in Mathematics, Science, and Engineering* (Reidel, Dordrecht, 1980). In this amazing volume one has analysis, approximation theory, operator theory, and detailed studies of a vast collection of variational problems.

A. M. Arthurs *Complementary Variational Principles* (Oxford U.P., Oxford, 1980). A rather special work describing ways of finding both upper and lower bounds. See also Weinstein and Stenger.

L. Pars *A Treatise on Analytical Mechanics* (John Wiley, New York, 1965). The title describes this book very well: it is a masterful presentation of the subject with copious fascinating problems worked out in leisurely detail. Strongly recommended!.

C. Lanczos *The Variational Principles of Mechanics* (University of Toronto Press, Toronto, 1970). This is mechanics set to music. It is a grand view of the mathematical and physical ideas.

G. Hadley *Nonlinear and dynamic programming* (Addison-Wesley, Reading MA, 1964). This book is a classic text. On p406 the elementary problem of calculus of variations is recast as a problem in dynamic programming. See also the excellent books by Richard Bellman on dynamic programming, optimization, and control theory.